Lines and Angles

Exercise: 6.1 (Page No: 96)

1. In Fig. 6.13, lines AB and CD intersect at O. If ∠AOC +∠BOE = 70° and ∠BOD = 40°, find ∠BOE and reflex ∠COE.

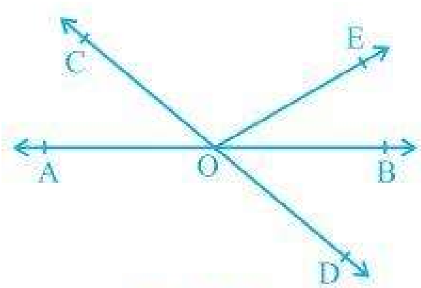


Fig. 6.13

Solution:
From the diagram, we have
(∠AOC +∠BOE +∠COE) and (∠COE +∠BOD +∠BOE) forms a straight line.
So, ∠AOC+∠BOE +∠COE = ∠COE +∠BOD+∠BOE = 180°
Now, by putting the values of ∠AOC + ∠BOE = 70° and ∠BOD = 40° we get
∠COE = 110° and ∠BOE = 30°
So, reflex ∠COE = 360° – 110° = 250°

2. In Fig. 6.14, lines XY and MN intersect at O. If ∠POY = 90° and a : b = 2 : 3, find c.

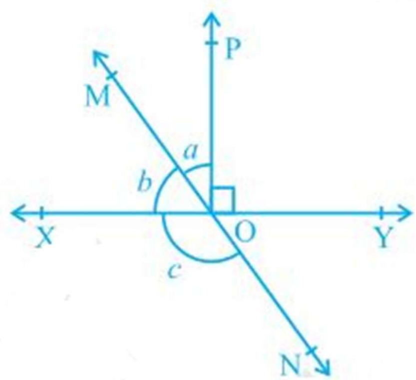


Fig. 6.14

Solution:
We know that the sum of linear pair is always equal to 180°
So,
∠POY +a +b = 180°
Putting the value of ∠POY = 90° (as given in the question), we get,

$a+b = 90°$
Now, it is given that a:b = 2:3, so
Let a be 2x and b be 3x
$\therefore 2x+3x = 90°$
Solving this, we get
$5x = 90°$
So, $x = 18°$
$\therefore a = 2\times18° = 36°$
Similarly, b can be calculated, and the value will be
$b = 3\times18° = 54°$
From the diagram, b+c also forms a straight angle, so
$b+c = 180°$
$c+54° = 180°$
$\therefore c = 126°$

3. In Fig. 6.15, ∠PQR = ∠PRQ, then prove that ∠PQS = ∠PRT.

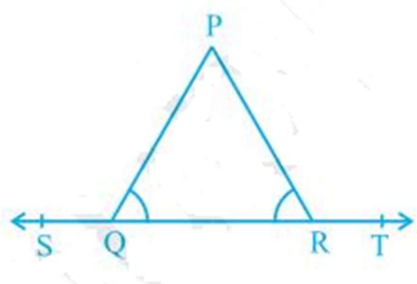


Fig. 6.15

Solution:
Since ST is a straight line, so
$\angle PQS+\angle PQR = 180°$ (linear pair) and
$\angle PRT+\angle PRQ = 180°$ (linear pair)
Now, $\angle PQS + \angle PQR = \angle PRT+\angle PRQ = 180°$
Since $\angle PQR = \angle PRQ$ (as given in the question)
$\angle PQS = \angle PRT$. (Hence proved).

4. In Fig. 6.16, if x+y = w+z, then prove that AOB is a line.

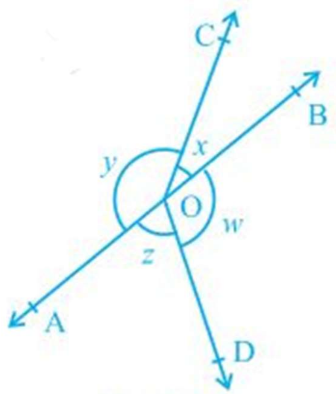


Fig. 6.16

Solution:
To prove AOB is a straight line, we will have to prove x+y is a linear pair
i.e. $x+y = 180°$
We know that the angles around a point are 360°, so
$x+y+w+z = 360°$
In the question, it is given that,
$x+y = w+z$
So, $(x+y)+(x+y) = 360°$
$2(x+y) = 360°$
$\therefore (x+y) = 180°$ (Hence proved).

5. In Fig. 6.17, POQ is a line. Ray OR is perpendicular to line PQ. OS is another ray lying between rays OP and OR. Prove that $\angle ROS = \frac{1}{2} (\angle QOS - \angle POS)$.

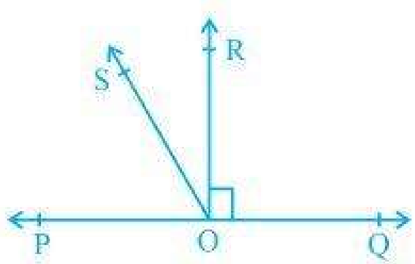


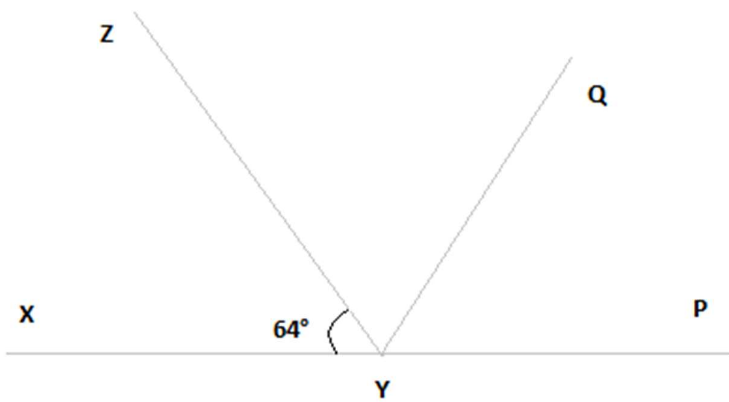
Fig. 6.17

Solution:
In the question, it is given that ($OR \perp PQ$) and $\angle POQ = 180°$
We can write it as $\angle ROP = \angle ROQ = 90^0$
We know that
$\angle ROP = \angle ROQ$
It can be written as

$\angle POS + \angle ROS = \angle ROQ$
$\angle POS + \angle ROS = \angle QOS - \angle ROS$
$\angle SOR + \angle ROS = \angle QOS - \angle POS$
So we get
$2\angle ROS = \angle QOS - \angle POS$
Or, $\angle ROS = 1/2\ (\angle QOS - \angle POS)$(Hence proved).

6. It is given that $\angle XYZ = 64°$ and XY is produced to point P. Draw a figure from the given information. If ray YQ bisects $\angle ZYP$, find $\angle XYQ$ and reflex $\angle QYP$.

Solution:



Here, XP is a straight line
So, $\angle XYZ + \angle ZYP = 180°$
Putting the value of $\angle XYZ = 64°$, we get
$64° + \angle ZYP = 180°$
$\therefore \angle ZYP = 116°$
From the diagram, we also know that $\angle ZYP = \angle ZYQ + \angle QYP$
Now, as YQ bisects $\angle ZYP$,
$\angle ZYQ = \angle QYP$
Or, $\angle ZYP = 2\angle ZYQ$
$\therefore \angle ZYQ = \angle QYP = 58°$
Again, $\angle XYQ = \angle XYZ + \angle ZYQ$
By putting the value of $\angle XYZ = 64°$ and $\angle ZYQ = 58°$, we get.
$\angle XYQ = 64° + 58°$
Or, $\angle XYQ = 122°$
Now, reflex $\angle QYP = 180° + XYQ$
We computed that the value of $\angle XYQ = 122°$.
So,
$\angle QYP = 180° + 122°$ $\therefore \angle QYP = 302°$

Exercise: 6.2 (Page No: 103)

1. In Fig. 6.28, find the values of x and y and then show that AB || CD.

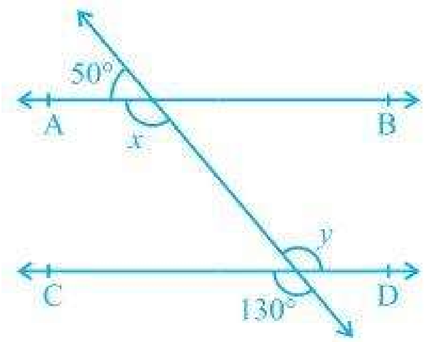


Fig. 6.28

Solution:

We know that a linear pair is equal to 180°.
So, x+50° = 180°
∴ x = 130°
We also know that vertically opposite angles are equal.
So, y = 130°
In two parallel lines, the alternate interior angles are equal. In this,
x = y = 130°
This proves that alternate interior angles are equal, so AB || CD.

2. In Fig. 6.29, if AB || CD, CD || EF and y : z = 3 : 7, find x.

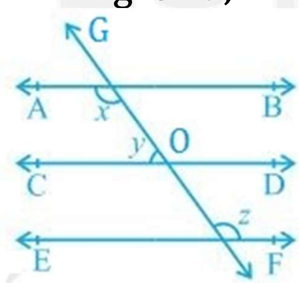


Fig. 6.29

Solution:

It is known that AB || CD and CD||EF
As the angles on the same side of a transversal line sum up to 180°,
x + y = 180° —–(i)
Also,
∠O = z (Since they are corresponding angles)
and, y +∠O = 180° (Since they are a linear pair)
So, y+z = 180°

Now, let y = 3w and hence, z = 7w (As y : z = 3 : 7)

∴ 3w+7w = 180°

Or, 10 w = 180°

So, w = 18°

Now, y = 3×18° = 54°

and, z = 7×18° = 126°

Now, angle x can be calculated from equation (i)

x+y = 180°

Or, x+54° = 180°

∴ x = 126°

3. In Fig. 6.30, if AB || CD, EF ⊥ CD and ∠GED = 126°, find ∠AGE, ∠GEF and ∠FGE.

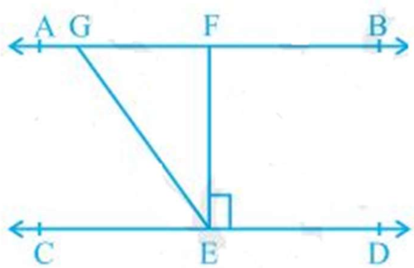


Fig. 6.30

Solution:

Since AB || CD, GE is a transversal.

It is given that ∠GED = 126°

So, ∠GED = ∠AGE = 126° (As they are alternate interior angles)

Also,

∠GED = ∠GEF +∠FED

As EF⊥ CD, ∠FED = 90°

∴ ∠GED = ∠GEF+90°

Or, ∠GEF = 126° – 90° = 36°

Again, ∠FGE +∠GED = 180° (Transversal)

Putting the value of ∠GED = 126°, we get

∠FGE = 54°

So,

∠AGE = 126°

∠GEF = 36° and

∠FGE = 54°

4. In Fig. 6.31, if PQ || ST, ∠PQR = 110° and ∠RST = 130°, find ∠QRS.

[Hint : Draw a line parallel to ST through point R.]

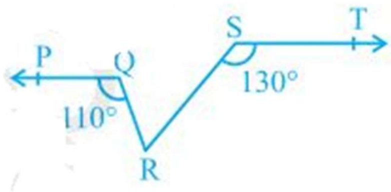
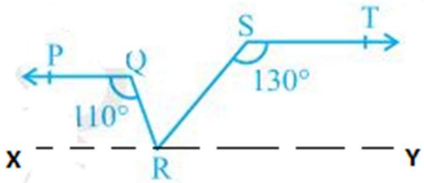


Fig. 6.31

Solution:

First, construct a line XY parallel to PQ.



We know that the angles on the same side of the transversal is equal to 180°.

So, ∠PQR+∠QRX = 180°

Or, ∠QRX = 180°-110°

∴ ∠QRX = 70°

Similarly,

∠RST +∠SRY = 180°

Or, ∠SRY = 180°- 130°

∴ ∠SRY = 50°

Now, for the linear pairs on the line XY-

∠QRX+∠QRS+∠SRY = 180°

Putting their respective values, we get

∠QRS = 180° – 70° – 50°

Hence, ∠QRS = 60°

5. In Fig. 6.32, if AB || CD, ∠APQ = 50° and ∠PRD = 127°, find x and y.

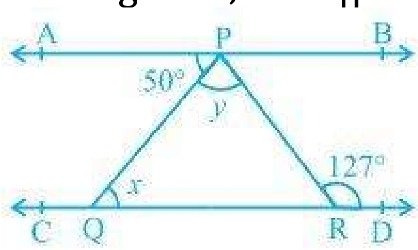


Fig. 6.32

Solution:

From the diagram,
∠APQ = ∠PQR (Alternate interior angles)
Now, putting the value of ∠APQ = 50° and ∠PQR = x, we get
x = 50°
Also,
∠APR = ∠PRD (Alternate interior angles)
Or, ∠APR = 127° (As it is given that ∠PRD = 127°)
We know that
∠APR = ∠APQ+∠QPR
Now, putting values of ∠QPR = y and ∠APR = 127°, we get
127° = 50°+ y
Or, y = 77°
Thus, the values of x and y are calculated as:
x = 50° and y = 77°

6. In Fig. 6.33, PQ and RS are two mirrors placed parallel to each other. An incident ray AB strikes the mirror PQ at B, the reflected ray moves along the path BC and strikes the mirror RS at C and again reflects back along CD. Prove that AB || CD.

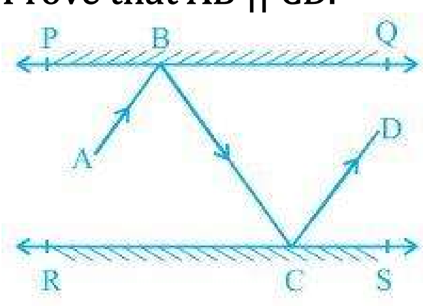
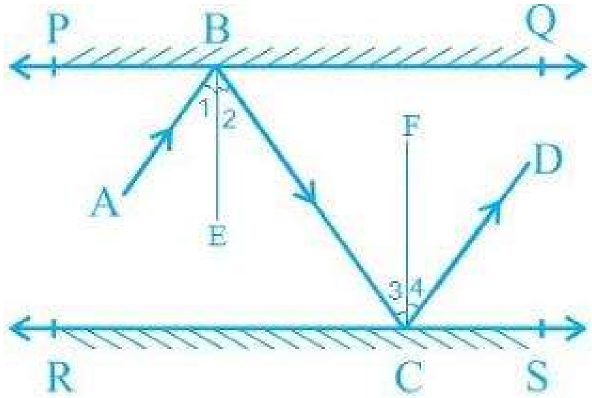


Fig. 6.33

Solution:
First, draw two lines, BE and CF, such that BE ⊥ PQ and CF ⊥ RS.
Now, since PQ || RS,
So, BE || CF



We know that,
Angle of incidence = Angle of reflection (By the law of reflection)
So,
$\angle 1 = \angle 2$ and
$\angle 3 = \angle 4$
We also know that alternate interior angles are equal. Here, BE ⊥ CF and the transversal line BC cuts them at B and C
So, $\angle 2 = \angle 3$ (As they are alternate interior angles)
Now, $\angle 1 + \angle 2 = \angle 3 + \angle 4$
Or, $\angle ABC = \angle DCB$
So, AB || CD (alternate interior angles are equal)

Exercise: 6.3 (Page No: 107)

1. In Fig. 6.39, sides QP and RQ of ΔPQR are produced to points S and T,respectively. If $\angle SPR = 135°$ and $\angle PQT = 110°$, find $\angle PRQ$.

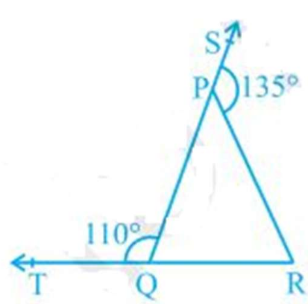


Fig. 6.39

Solution:
It is given the TQR is a straight line, and so, the linear pairs (i.e. $\angle TQP$ and $\angle PQR$) will add up to 180°
So, $\angle TQP + \angle PQR = 180°$
Now, putting the value of $\angle TQP = 110°$, we get

∠PQR = 70°

Consider the ΔPQR,

Here, the side QP is extended to S, and so ∠SPR forms the exterior angle.

Thus, ∠SPR (∠SPR = 135°) is equal to the sum of interior opposite angles. (Triangle property)

Or, ∠PQR +∠PRQ = 135°

Now, putting the value of ∠PQR = 70°, we get

∠PRQ = 135°-70°

Hence, ∠PRQ = 65°

2. In Fig. 6.40, ∠X = 62°, ∠XYZ = 54°. If YO and ZO are the bisectors of ∠XYZ and ∠XZY, respectively of Δ XYZ, find ∠OZY and ∠YOZ.

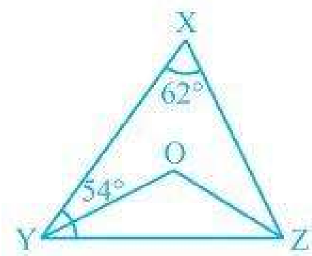


Fig. 6.40

Solution:

We know that the sum of the interior angles of the triangle.

So, ∠X +∠XYZ +∠XZY = 180°

Putting the values as given in the question, we get

62°+54° +∠XZY = 180°

Or, ∠XZY = 64°

Now, we know that ZO is the bisector, so

∠OZY = ½ ∠XZY

∴ ∠OZY = 32°

Similarly, YO is a bisector, so

∠OYZ = ½ ∠XYZ

Or, ∠OYZ = 27° (As ∠XYZ = 54°)

Now, as the sum of the interior angles of the triangle,

∠OZY +∠OYZ +∠O = 180°

Putting their respective values, we get

∠O = 180°-32°-27°

Hence, ∠O = 121°

3. In Fig. 6.41, if AB || DE, ∠BAC = 35° and ∠CDE = 53°, find ∠DCE.

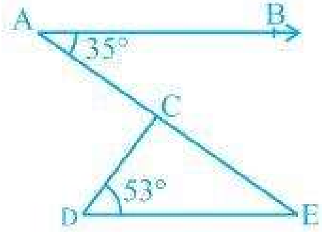


Fig. 6.41

Solution:

We know that AE is a transversal since AB || DE

Here ∠BAC and ∠AED are alternate interior angles.

Hence, ∠BAC = ∠AED

It is given that ∠BAC = 35°

∠AED = 35°

Now consider the triangle CDE. We know that the sum of the interior angles of a triangle is 180°.

∴ ∠DCE+∠CED+∠CDE = 180°

Putting the values, we get

∠DCE+35°+53° = 180°

Hence, ∠DCE = 92°

4. In Fig. 6.42, if lines PQ and RS intersect at point T, such that ∠PRT = 40°, ∠RPT = 95° and ∠TSQ = 75°, find ∠SQT.

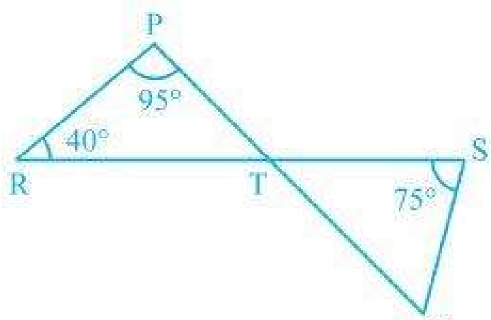


Fig. 6.42

Solution:

Consider triangle PRT.

∠PRT +∠RPT + ∠PTR = 180°

So, ∠PTR = 45°

Now ∠PTR will be equal to ∠STQ as they are vertically opposite angles.

So, ∠PTR = ∠STQ = 45°

Again, in triangle STQ,

∠TSQ +∠PTR + ∠SQT = 180°

Solving this, we get

$74° + 45° + \angle SQT = 180°$

$\angle SQT = 60°$

5. In Fig. 6.43, if PQ ⊥ PS, PQ || SR, ∠SQR = 28° and ∠QRT = 65°, then find the values of x and y.

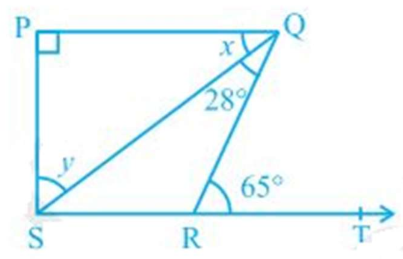


Fig. 6.43

Solution:

$x + \angle SQR = \angle QRT$ (As they are alternate angles since QR is transversal)

So, $x+28° = 65°$

$\therefore x = 37°$

It is also known that alternate interior angles are the same, and so

$\angle QSR = x = 37°$

Also, now

$\angle QRS + \angle QRT = 180°$ (As they are a Linear pair)

Or, $\angle QRS+65° = 180°$

So, $\angle QRS = 115°$

Using the angle sum property in Δ SPQ,

$\angle SPQ + x + y = 180°$

Putting their respective values, we get

$90°+37° + y = 180°$

$y = 180^0 – 127^0 = 53^0$

Hence, $y = 53°$

6. In Fig. 6.44, the side QR of ΔPQR is produced to a point S. If the bisectors of ∠PQR and ∠PRS meet at point T, then prove that ∠QTR = ½ ∠QPR.

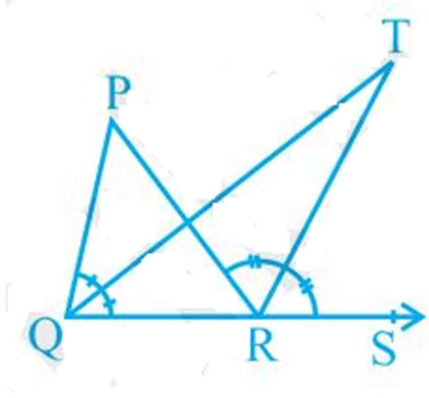


Fig. 6.44

Solution:

Consider the ΔPQR. ∠PRS is the exterior angle, and ∠QPR and ∠PQR are the interior angles.

So, ∠PRS = ∠QPR+∠PQR (According to triangle property)

Or, ∠PRS -∠PQR = ∠QPR ————(i)

Now, consider the ΔQRT,

∠TRS = ∠TQR+∠QTR

Or, ∠QTR = ∠TRS-∠TQR

We know that QT and RT bisect ∠PQR and ∠PRS, respectively.

So, ∠PRS = 2 ∠TRS and ∠PQR = 2∠TQR

Now, ∠QTR = ½ ∠PRS – ½∠PQR

Or, ∠QTR = ½ (∠PRS -∠PQR)

From (i), we know that ∠PRS -∠PQR = ∠QPR

So, ∠QTR = ½ ∠QPR (hence proved).

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